1. **Send money.** When they send out their fundraising letter, a philanthropic organization typically gets a return from about 5% of the people on their mailing list. To see what the response rate might be for future appeals, they did a simulation using samples of size 20, 50, 100, and 200. For each sample size, they simulated 1000 mailings with success rate \( p = 0.05 \) and constructed the histogram of the 1000 sample proportions, shown below. Explain how these histograms demonstrate what the Central Limit Theorem says about the sampling distribution model for sample proportions. Be sure to talk about shape, center, and spread.

2. **Character recognition.** An automatic character recognition device can successfully read about 85% of handwritten credit card applications. To estimate what might happen when this device reads a stack of applications, the company did a simulation using samples of size 20, 50, 75, and 100. For each sample size, they simulated 1000 samples with success rate \( p = 0.85 \) and constructed the histogram of the 1000 sample proportions, shown here. Explain how these histograms demonstrate what the Central Limit Theorem says about the sampling distribution model for sample proportions. Be sure to talk about shape, center, and spread.

3. **Send money, again.** The philanthropic organization in Exercise 1 expects about a 5% success rate when they send fundraising letters to the people on their mailing list. In Exercise 1 you looked at the histograms showing distributions of sample proportions from 1000 simulated mailings for samples of size 20, 50, 100, and 200. The sample statistics from each simulation were as follows:

<table>
<thead>
<tr>
<th>n</th>
<th>mean</th>
<th>st. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0497</td>
<td>0.0479</td>
</tr>
<tr>
<td>50</td>
<td>0.0516</td>
<td>0.0309</td>
</tr>
<tr>
<td>100</td>
<td>0.0497</td>
<td>0.0215</td>
</tr>
<tr>
<td>200</td>
<td>0.0501</td>
<td>0.0152</td>
</tr>
</tbody>
</table>

a) According to the Central Limit Theorem, what should the theoretical mean and standard deviations be for these sample sizes?
b) How close are these theoretical values to what was observed in these simulations?
c) Looking at the histograms in Exercise 1, at what sample size would you be comfortable using the Normal model as an approximation for the sampling distribution?
d) What does the Success/Failure Condition say about the choice you made in part c?
4. Character recognition, again. The automatic character recognition device discussed in Exercise 2 successfully reads about 85% of handwritten credit card applications. In Exercise 2 you looked at the histograms showing distributions of sample proportions from 1000 simulated samples of size 20, 50, 75, and 100. The sample statistics from each simulation were as follows:

<table>
<thead>
<tr>
<th>n</th>
<th>mean</th>
<th>st. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.8481</td>
<td>0.0803</td>
</tr>
<tr>
<td>50</td>
<td>0.8507</td>
<td>0.0509</td>
</tr>
<tr>
<td>75</td>
<td>0.8481</td>
<td>0.0406</td>
</tr>
<tr>
<td>100</td>
<td>0.8488</td>
<td>0.0354</td>
</tr>
</tbody>
</table>

a) According to the Central Limit Theorem, what should the theoretical mean and standard deviations be for these sample sizes?
b) How close are those theoretical values to what was observed in these simulations?
c) Looking at the histograms in Exercise 2, at what sample size would you be comfortable using the Normal model as an approximation for the sampling distribution?
d) What does the Success/Failure Condition say about the choice you made in part c?

5. Coin tosses. In a large class of introductory Statistics students, the professor has each person toss a coin 16 times and calculate the proportion of heads. The students then report their results, and the professor plots a histogram of these several proportions.
a) What shape would you expect this histogram to be? Why?
b) Where do you expect the histogram to be centered?
c) How much variability would you expect among these proportions?
d) Explain why a Normal model should not be used here.

6. M&M’s. The candy company claims that 10% of the M&M’s it produces are green. Suppose that the candies are packaged at random in small bags containing about 50 M&M’s. A class of elementary school students learning about percentages opens several bags, counts the various colors of the candies, and calculates the proportion that are green.
a) If we plot a histogram showing the proportions of green candies in the various bags, what shape would you expect it to have?
b) Can that histogram be approximated by a Normal model? Explain.
c) Where should the center of the histogram be?
d) What should the standard deviation of the proportion be?

7. More coins. Suppose the class in Exercise 5 repeats the coin-tossing experiment.
a) The students toss the coins 25 times each. Use the 68–95–99.7 Rule to describe the sampling distribution model.
b) Confirm that you can use a Normal model here.
c) They increase the number of tosses to 64 each. Draw and label the appropriate sampling distribution model. Check the appropriate conditions to justify your model.
d) Explain how the sampling distribution model changes as the number of tosses increases.

8. Bigger bag. Suppose the class in Exercise 6 buys bigger bags of candy, with 200 M&M’s each. Again the students calculate the proportion of green candies they find.
a) Explain why it’s appropriate to use a Normal model to describe the distribution of the proportion of green M&M’s they might expect.
b) Use the 68–95–99.7 Rule to describe how this proportion might vary from bag to bag.
c) How would this model change if the bags contained even more candies?

9. Just (un)lucky? One of the students in the introductory Statistics class in Exercise 7 claims to have tossed her coin 200 times and found only 42% heads. What do you think of this claim? Explain.

10. Too many green ones? In a really large bag of M&M’s, the students in Exercise 8 found 500 candies, and 12% of them were green. Is this an unusually large proportion of green M&M’s? Explain.

11. Speeding. State police believe that 70% of the drivers traveling on a major interstate highway exceed the speed limit. They plan to set up a radar trap and check the speeds of 80 cars.
a) Using the 68–95–99.7 Rule, draw and label the distribution of the proportion of these cars the police will observe speeding.
b) Do you think the appropriate conditions necessary for your analysis are met? Explain.

12. Smoking. Public health statistics indicate that 26.4% of American adults smoke cigarettes. Using the 68–95–99.7 Rule, describe the sampling distribution model for the proportion of smokers among a randomly selected group of 50 adults. Be sure to discuss your assumptions and conditions.

13. Vision. It is generally believed that nearsightedness affects about 12% of all children. A school district has registered 170 incoming kindergarten children.
a) Can you apply the Central Limit Theorem to describe the sampling distribution model for the sample proportion of children who are nearsighted? Check the conditions and discuss any assumptions you need to make.
b) Sketch and clearly label the sampling model, based on the 68–95–99.7 Rule.
c) How many of the incoming students might the school expect to be nearsighted? Explain.

14. Mortgages. In early 2007 the Mortgage Lenders Association reported that homeowners, hit hard by rising interest rates on adjustable-rate mortgages, were defaulting in record numbers. The foreclosure rate of 1.6% meant that millions of families were losing their homes. Suppose a large bank holds 1731 adjustable-rate mortgages.
a) Can you apply the Central Limit Theorem to describe the sampling distribution model for the sample proportion of foreclosures? Check the conditions and discuss any assumptions you need to make.
b) Sketch and clearly label the sampling model, based on the 68–95–99.7 Rule.
c) How many of these homeowners might the bank expect to default on their mortgages? Explain.
15. Loans. Based on past experience, a bank believes that 7% of the people who receive loans will not make payments on time. The bank has recently approved 200 loans.
   a) What are the mean and standard deviation of the proportion of clients in this group who may not make timely payments?
   b) What assumptions underlie your model? Are the conditions met? Explain.
   c) What’s the probability that over 10% of these clients will not make timely payments?

16. Contacts. Assume that 30% of students at a university wear contact lenses.
   a) We randomly pick 100 students. Let \( \hat{p} \) represent the proportion of students in this sample who wear contacts. What’s the appropriate model for the distribution of \( \hat{p} \)? Specify the name of the distribution, the mean, and the standard deviation. Be sure to verify that the conditions are met.
   b) What’s the approximate probability that more than one third of this sample wear contacts?

17. Back to school? Best known for its testing program, ACT, Inc., also compiles data on a variety of issues in education. In 2004 the company reported that the national college freshman-to-sophomore retention rate held steady at 74% over the previous four years. Consider random samples of 400 freshmen who took the ACT. Use the 68–95–99.7 Rule to describe the sampling distribution model for the percentage of those students we expect to return to that school for their sophomore years. Do you think the appropriate conditions are met?

18. Binge drinking. As we learned in Chapter 15, a national study found that 44% of college students engage in binge drinking (5 drinks at a sitting for men, 4 for women). Use the 68–95–99.7 Rule to describe the sampling distribution model for the proportion of students in a randomly selected group of 200 college students who engage in binge drinking. Do you think the appropriate conditions are met?

19. Back to school, again. Based on the 74% national retention rate described in Exercise 17, does a college where 522 of the 603 freshman returned the next year as sophomores have a right to brag that it has an unusually high retention rate? Explain.

20. Binge sample. After hearing of the national result that 44% of students engage in binge drinking (5 drinks at a sitting for men, 4 for women), a professor surveyed a random sample of 244 students at his college and found that 96 of them admitted to binge drinking in the past week. Should he be surprised at this result? Explain.

21. Polling. Just before a referendum on a school budget, a local newspaper polls 400 voters in an attempt to predict whether the budget will pass. Suppose that the budget actually has the support of 52% of the voters. What’s the probability the newspaper’s sample will lead them to predict defeat? Be sure to verify that the assumptions and conditions necessary for your analysis are met.

22. Seeds. Information on a packet of seeds claims that the germination rate is 92%. What’s the probability that more than 95% of the 160 seeds in the packet will germinate? Be sure to discuss your assumptions and check the conditions that support your model.

23. Apples. When a truckload of apples arrives at a packing plant, a random sample of 150 is selected and examined for bruises, discoloration, and other defects. The whole truckload will be rejected if more than 5% of the sample is unsatisfactory. Suppose that in fact 8% of the apples on the truck do not meet the desired standard. What’s the probability that the shipment will be accepted anyway?

24. Genetic defect. It’s believed that 4% of children have a gene that may be linked to juvenile diabetes. Researchers hoping to track 20 of these children for several years test 732 newborns for the presence of this gene. What’s the probability that they find enough subjects for their study?

25. Nonsmokers. While some nonsmokers do not mind being seated in a smoking section of a restaurant, about 60% of the customers demand a smoke-free area. A new restaurant with 120 seats is being planned. How many seats should be in the nonsmoking area in order to be very sure of having enough seating there? Comment on the assumptions and conditions that support your model, and explain what “very sure” means to you.

26. Meals. A restauranteur anticipates serving about 180 people on a Friday evening, and believes that about 20% of the patrons will order the chef’s steak special. How many steaks should he plan on serving in order to be pretty sure of having enough steaks on hand to meet customer demand? Justify your answer, including an explanation of what “pretty sure” means to you.

27. Sampling. A sample is chosen randomly from a population that can be described by a Normal model.
   a) What’s the sampling distribution model for the sample mean? Describe shape, center, and spread.
   b) If we choose a larger sample, what’s the effect on this sampling distribution model?

28. Sampling, part II. A sample is chosen randomly from a population that was strongly skewed to the left.
   a) Describe the sampling distribution model for the sample mean if the sample size is small.
   b) If we make the sample larger, what happens to the sampling distribution model’s shape, center, and spread?
   c) As we make the sample larger, what happens to the expected distribution of the data in the sample?

29. Waist size. A study measured the Waist Size of 250 men, finding a mean of 36.33 inches and a standard deviation of 4.02 inches. Here is a histogram of these measurements.

   ![Histogram of Waist Size](image)

   a) Describe the histogram of Waist Size.
b) To explore how the mean might vary from sample to sample, they simulated by drawing many samples of size 2, 5, 10, and 20, with replacement, from the 250 measurements. Here are histograms of the sample means for each simulation. Explain how these histograms demonstrate what the Central Limit Theorem says about the sampling distribution model for sample means.

b) Explain how these histograms demonstrate what the Central Limit Theorem says about the sampling distribution model for sample means. Be sure to talk about shape, center, and spread.

c) Comment on the “rule of thumb” that “With a sample size of at least 30, the sampling distribution of the mean is Normal”?

30. CEO compensation. In Chapter 5 we saw the distribution of the total compensation of the chief executive officers (CEOs) of the 800 largest U.S. companies (the Fortune 800). The average compensation (in thousands of dollars) is 10,307.31 and the standard deviation is 17,964.62. Here is a histogram of their annual compensations (in $1000):

a) Describe the histogram of Total Compensation.
A research organization simulated sample means by drawing samples of 30, 50, 100, and 200, with replacement, from the 800 CEOs. The histograms show the distributions of means for many samples of each size.

b) According to the Central Limit Theorem, what should the theoretical mean and standard deviation be for each of these sample sizes?

b) How close are the theoretical values to what was observed in the simulation?

c) Looking at the histograms in Exercise 29, at what sample size would you be comfortable using the Normal model as an approximation for the sampling distribution?

d) What about the shape of the distribution of Waist Size explains your choice of sample size in part c?

31. Waist size revisited. Researchers measured the Waist Sizes of 250 men in a study on body fat. The true mean and standard deviation of the Waist Sizes for the 250 men are 36.33 in and 4.019 inches, respectively. In Exercise 29 you looked at the histograms of simulations that drew samples of sizes 2, 5, 10, and 20 (with replacement). The summary statistics for these simulations were as follows:

<table>
<thead>
<tr>
<th>n</th>
<th>mean</th>
<th>st. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36.314</td>
<td>2.855</td>
</tr>
<tr>
<td>5</td>
<td>36.314</td>
<td>1.805</td>
</tr>
<tr>
<td>10</td>
<td>36.341</td>
<td>1.276</td>
</tr>
<tr>
<td>20</td>
<td>36.339</td>
<td>0.895</td>
</tr>
</tbody>
</table>

a) According to the Central Limit Theorem, what should the theoretical mean and standard deviation be for each of these sample sizes?

b) How close are the theoretical values to what was observed in the simulation?
32. **CEOs revisited.** In Exercise 30 you looked at the annual compensation for 800 CEOs, for which the true mean and standard deviation were (in thousands of dollars) 10,307.31 and 17,964.62, respectively. A simulation drew samples of sizes 30, 50, 100, and 200 (with replacement) from the total annual compensations of the Fortune 800 CEOs. The summary statistics for these simulations were as follows:

<table>
<thead>
<tr>
<th>n</th>
<th>mean</th>
<th>st. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10,251.73</td>
<td>3359.64</td>
</tr>
<tr>
<td>50</td>
<td>10,343.93</td>
<td>2483.84</td>
</tr>
<tr>
<td>100</td>
<td>10,329.94</td>
<td>1779.18</td>
</tr>
<tr>
<td>200</td>
<td>10,340.37</td>
<td>1250.79</td>
</tr>
</tbody>
</table>

a) According to the Central Limit Theorem, what should the theoretical mean and standard deviation be for each of these sample sizes?  
b) How close are the theoretical values to what was observed from the simulation?  
c) Looking at the histograms in Exercise 30, at what sample size would you be comfortable using the Normal model as an approximation for the sampling distribution?  
d) What about the shape of the distribution of Total Compensation explains your answer in part c?  

33. **GPAs.** A college’s data about the incoming freshmen indicates that the mean of their high school GPAs was 3.4, with a standard deviation of 0.35; the distribution was roughly mound-shaped and only slightly skewed. The students are randomly assigned to freshman writing seminars in groups of 25. What might the mean GPA of one of these seminar groups be? Describe the appropriate sampling distribution model—shape, center, and spread—with attention to assumptions and conditions. Make a sketch using the 68–95–99.7 Rule.

34. **Home values.** Assessment records indicate that the value of homes in a small city is skewed right, with a mean of $140,000 and standard deviation of $60,000. To check the accuracy of the assessment data, officials plan to conduct a detailed appraisal of 100 homes selected at random. Using the 68–95–99.7 Rule, draw and label an appropriate sampling model for the mean value of homes selected.

35. **Lucky Spot?** A reporter working on a story about the New York lottery contacted one of the authors of this book, wanting help analyzing data to see if some ticket sales outlets were more likely to produce winners. His data for each of the 966 New York lottery outlets are graphed below; the scatterplot shows the ratio TotalPaid/TotalSales vs. TotalSales for the state’s “instant winner” games for all of 2007.

The reporter thinks that by identifying the outlets with the highest fraction of bets paid out, players might be able to increase their chances of winning. (Typically—but not always—instant winners are paid immediately (instantly) at the store at which they are purchased. However, the fact that tickets may be scratched off and then cashed in at any outlet may account for some outlets paying out more than they take in. The few with very low payouts may be on interstate highways where players may purchase cards but then leave.)

a) Explain why the plot has this funnel shape.  
b) Explain why the reporter’s idea wouldn’t have worked anyway.

36. **Safe cities.** Allstate Insurance Company identified the 10 safest and 10 least-safe U.S. cities from among the 200 largest cities in the United States, based on the mean number of years drivers went between automobile accidents. The cities on both lists were all smaller than the 10 largest cities. Using facts about the sampling distribution model of the mean, explain why this is not surprising.

37. **Pregnancy.** Assume that the duration of human pregnancies can be described by a Normal model with mean 266 days and standard deviation 16 days.

a) What percentage of pregnancies should last between 270 and 280 days?  
b) At least how many days should the longest 25% of all pregnancies last?  
c) Suppose a certain obstetrician is currently providing prenatal care to 60 pregnant women. Let represent the mean length of their pregnancies. According to the Central Limit Theorem, what’s the distribution of this sample mean, ? Specify the model, mean, and standard deviation.  
d) What’s the probability that the mean duration of these patients’ pregnancies will be less than 260 days?

38. **Rainfall.** Statistics from Cornell’s Northeast Regional Climate Center indicate that Ithaca, NY, gets an average of 35.4” of rain each year, with a standard deviation of 4.2”. Assume that a Normal model applies.

a) During what percentage of years does Ithaca get more than 40” of rain?  
b) Less than how much rain falls in the driest 20% of all years?  
c) A Cornell University student is in Ithaca for 4 years. Let represent the mean amount of rain for those 4 years. Describe the sampling distribution model of this sample mean, .  
d) What’s the probability that those 4 years average less than 30” of rain?

39. **Pregnant again.** The duration of human pregnancies may not actually follow the Normal model described in Exercise 37.

a) Explain why it may be somewhat skewed to the left.  
b) If the correct model is in fact skewed, does that change your answers to parts a, b, and c of Exercise 37? Explain why or why not for each.

40. **At work.** Some business analysts estimate that the length of time people work at a job has a mean of 6.2 years and a standard deviation of 4.5 years.
41. **Dice and dollars.** You roll a die, winning nothing if the number of spots is even, $1 for a 2 or a 4, and $10 for a 6.
   a) Find the expected value and standard deviation of your prospective winnings.
   b) You play twice. Find the mean and standard deviation of your total winnings.
   c) You play 40 times. What’s the probability that you win at least $100?
42. **New game.** You pay $10 and roll a die. If you get a 6, you win $50. If not, you get to roll again. If you get a 6 this time, you get your $10 back.
   a) Create a probability model for this game.
   b) Find the expected value and standard deviation of your average winnings.
   c) You play this game five times. Find the expected value and standard deviation of your average winnings.
   d) 100 people play this game. What’s the probability that the mean of their total winnings is at least $100?
43. **AP Stats 2006.** The College Board reported the score distribution shown in the table for all students who took the 2006 AP Statistics exam.

<table>
<thead>
<tr>
<th>Score</th>
<th>Percent of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>12.6</td>
</tr>
<tr>
<td>4</td>
<td>22.2</td>
</tr>
<tr>
<td>3</td>
<td>25.3</td>
</tr>
<tr>
<td>2</td>
<td>18.3</td>
</tr>
<tr>
<td>1</td>
<td>21.6</td>
</tr>
</tbody>
</table>

   a) Find the mean and standard deviation of the scores.
   b) If we select a random sample of 40 AP Statistics students, would you expect their scores to follow a Normal model? Explain.
   c) Consider the mean scores of random samples of 40 AP Statistics students. Describe the sampling model for these means (shape, center, and spread).
45. **AP Stats 2006, again.** An AP Statistics teacher had 63 students preparing to take the AP exam discussed in Exercise 43. Though they were obviously not a random sample, he considered his students to be “typical” of all the national students. What’s the probability that his students will achieve an average score of at least 3?
46. **Joining the museum.** One of the museum’s phone volunteers sets a personal goal of getting an average donation of at least $100 from the new members she enrolls during the membership drive. If she gets 80 new members and they can be considered a random sample of all the museum’s members, what is the probability that she can achieve her goal?
47. **Pollution.** Carbon monoxide (CO) emissions for a certain kind of car vary with mean 2.9 g/mi and standard deviation 0.4 g/mi. A company has 80 of these cars in its fleet. Let $\bar{y}$ represent the mean CO level for the company’s fleet.
   a) What’s the approximate model for the distribution of $\bar{y}$? Explain.
   b) Estimate the probability that $\bar{y}$ is between 3.0 and 3.1 g/mi.
   c) There is only a 5% chance that the mean CO level is greater than what value?
48. **Potato chips.** The weight of potato chips in a medium-size bag is stated to be 10 ounces. The amount that the packaging machine puts in these bags is believed to have a Normal model with mean 10.2 ounces and standard deviation 0.12 ounces.
   a) What fraction of all bags sold are underweight?
   b) Some of the chips are sold in “bargain packs” of 3 bags. What’s the probability that none of the 3 is underweight?
   c) What’s the probability that the mean weight of the 3 bags is below the stated amount?
   d) What’s the probability that the mean weight of a 24-bag case of potato chips is below 10 ounces?
49. **Tips.** A waiter believes the distribution of his tips has a model that is slightly skewed to the right, with a mean of $9.60 and a standard deviation of $5.40.
   a) Explain why you cannot determine the probability that a given party will tip him at least $20.
   b) Can you estimate the probability that the next 4 parties will tip an average of at least $15? Explain.
   c) Is it likely that his 10 parties today will tip an average of at least $15? Explain.
50. **Groceries.** A grocery store’s receipts show that Sunday customer purchases have a skewed distribution with a mean of $32 and a standard deviation of $20.
   a) Explain why you cannot determine the probability that the next Sunday customer will spend at least $40.
   b) Can you estimate the probability that the next 10 Sunday customers will spend an average of at least $40? Explain.
   c) Is it likely that the next 50 Sunday customers will spend an average of at least $40? Explain.
51. More tips. The waiter in Exercise 49 usually waits on about 40 parties over a weekend of work.
   a) Estimate the probability that he will earn at least $500 in tips.
   b) How much does he earn on the best 10% of such weekends?

52. More groceries. Suppose the store in Exercise 50 had 312 customers this Sunday.
   a) Estimate the probability that the store’s revenues were at least $10,000.
   b) If, on a typical Sunday, the store serves 312 customers, how much does the store take in on the worst 10% of such days?

53. IQs. Suppose that IQs of East State University’s students can be described by a Normal model with mean 130 and standard deviation 8 points. Also suppose that IQs of students from West State University can be described by a Normal model with mean 120 and standard deviation 10.
   a) We select a student at random from East State. Find the probability that this student’s IQ is at least 125 points.
   b) We select a student at random from each school. Find the probability that the East State student’s IQ is at least 5 points higher than the West State student’s IQ.
   c) We select 3 West State students at random. Find the probability that this group’s average IQ is at least 125 points.
   d) We also select 3 East State students at random. What’s the probability that their average IQ is at least 5 points higher than the average for the 3 West Staters?

54. Milk. Although most of us buy milk by the quart or gallon, farmers measure daily production in pounds. Ayrshire cows average 47 pounds of milk a day, with a standard deviation of 5 pounds. Assume that Normal models describe milk production for these breeds.
   a) We select an Ayrshire at random. What’s the probability that she averages more than 50 pounds of milk a day?
   b) What’s the probability that a randomly selected Ayrshire gives more milk than a randomly selected Jersey?
   c) A farmer has 20 Jerseys. What’s the probability that the average production for this small herd exceeds 45 pounds of milk a day?
   d) A neighboring farmer has 10 Ayrshires. What’s the probability that his herd average is at least 5 pounds higher than the average for part c’s Jersey herd?