Most statistics packages offer functions that compute Binomial probabilities, and many offer functions for Geometric probabilities as well. Some technology solutions automatically use the Normal approximation for the Binomial when the exact calculations become unmanageable.

**Exercises**

1. **Bernoulli.** Do these situations involve Bernoulli trials? Explain.
   a) We roll 50 dice to find the distribution of the number of spots on the faces.
   b) How likely is it that in a group of 120 the majority may have Type A blood, given that Type A is found in 43% of the population?
   c) We deal 7 cards from a deck and get all hearts. How likely is that?
   d) We wish to predict the outcome of a vote on the school budget, and poll 500 of the 3000 likely voters to see how many favor the proposed budget.
   e) A company realizes that about 10% of its packages are not being sealed properly. In a case of 24, is it likely that more than 3 are unsealed?

2. **Bernoulli 2.** Do these situations involve Bernoulli trials? Explain.
   a) You are rolling 5 dice and need to get at least two 6’s to win the game.
   b) We record the distribution of eye colors found in a group of 500 people.
   c) A manufacturer recalls a doll because about 3% have buttons that are not properly attached. Customers return 37 of these dolls to the local toy store. Is the manufacturer likely to find any dangerous buttons?
   d) A city council of 11 Republicans and 8 Democrats picks a committee of 4 at random. What’s the probability they choose all Democrats?
   e) A 2002 Rutgers University study found that 74% of high school students have cheated on a test at least once. Your local high school principal conducts a survey in homerooms and gets responses that admit to cheating from 322 of the 481 students.

3. **Simulating the model.** Think about the Tiger Woods picture search again. You are opening boxes of cereal one at a time looking for his picture, which is in 20% of the boxes. You want to know how many boxes you might have to open in order to find Tiger.
   a) Describe how you would simulate the search for Tiger using random numbers.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that you might find your first picture of Tiger in the first box, the second, etc.
   d) Calculate the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

4. **Simulation II.** You are one space short of winning a child’s board game and must roll a 1 on a die to claim victory. You want to know how many rolls it might take.
   a) Describe how you would simulate rolling the die until you get a 1.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that you might win on the first roll, the second, the third, etc.
   d) Calculate the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

5. **Tiger again.** Let’s take one last look at the Tiger Woods picture search. You know his picture is in 20% of the cereal boxes. You buy five boxes to see how many pictures of Tiger you might get.
   a) Describe how you would simulate the number of pictures of Tiger you might find in five boxes of cereal.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that you get no pictures of Tiger, 1 picture, 2 pictures, etc.
   d) Find the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

6. **Seatbelts.** Suppose 75% of all drivers always wear their seatbelts. Let’s investigate how many of the drivers might be belted among five cars waiting at a traffic light.
   a) Describe how you would simulate the number of seatbelt-wearing drivers among the five cars.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that there are no belted drivers, exactly one, two, etc.
   d) Find the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

7. **On time.** A Department of Transportation report about air travel found that, nationwide, 76% of all flights are on time. Suppose you are at the airport and your flight is one of 50 scheduled to take off in the next two hours. Can you consider these departures to be Bernoulli trials? Explain.
8. **Lost luggage.** A Department of Transportation report about air travel found that airlines misplace about 5 bags per 1000 passengers. Suppose you are traveling with a group of people who have checked 22 pieces of luggage on your flight. Can you consider the fate of these bags to be Bernoulli trials? Explain.

9. **Hoops.** A basketball player has made 80% of his foul shots during the season. Assuming the shots are independent, find the probability that in tonight’s game he a) misses for the first time on his fifth attempt. b) makes his first basket on his fourth shot. c) makes his first basket on one of his first 3 shots.

10. **Chips.** Suppose a computer chip manufacturer rejects 2% of the chips produced because they fail presale testing. a) What’s the probability that the fifth chip you test is the first bad one you find? b) What’s the probability you find a bad one within the first 10 you examine?

11. **More hoops.** For the basketball player in Exercise 9, what’s the expected number of shots until he misses?

12. **Chips ahoi.** For the computer chips described in Exercise 10, how many do you expect to test before finding a bad one?

13. **Customer center operator.** Raaj works at the customer service call center of a major credit card bank. Cardholders call for a variety of reasons, but regardless of their reason for calling, if they hold a platinum card, Raaj is instructed to offer them a double-miles promotion. About 10% of all cardholders hold platinum cards, and about 50% of those will take the double-miles promotion. On average, how many calls will Raaj have to take before finding the first cardholder to take the double-miles promotion?

14. **Cold calls.** Justine works for an organization committed to raising money for Alzheimer’s research. From past experience, the organization knows that about 20% of all potential donors will agree to give something if contacted by phone. They also know that of all people donating, about 5% will give $100 or more. On average, how many potential donors will Justine have to contact until she gets her first $100 donor?

15. **Blood.** Only 4% of people have Type AB blood. a) On average, how many donors must be checked to find someone with Type AB blood? b) What’s the probability that there is a Type AB donor among the first 5 people checked? c) What’s the probability that the first Type AB donor will be found among the first 6 people? d) What’s the probability that we won’t find a Type AB donor before the 10th person?

16. **Colorblindness.** About 8% of males are colorblind. A researcher needs some colorblind subjects for an experiment and begins checking potential subjects. a) On average, how many men should the researcher expect to check to find one who is colorblind? b) What’s the probability that she won’t find anyone colorblind among the first 4 men she checks?

17. **Lefties.** Assume that 13% of people are left-handed. If we select 5 people at random, find the probability of each outcome described below.

   a) The first lefty is the fifth person chosen.
   b) There are some lefties among the 5 people.
   c) The first lefty is the second or third person.
   d) There are exactly 3 lefties in the group.
   e) There are at least 3 lefties in the group.
   f) There are no more than 3 lefties in the group.

18. **Arrows.** An Olympic archer is able to hit the bull’s-eye 80% of the time. Assume each shot is independent of the others. If she shoots 6 arrows, what’s the probability of each of the following results?

   a) Her first bull’s-eye comes on the third arrow.
   b) She misses the bull’s-eye at least once.
   c) Her first bull’s-eye comes on the fourth or fifth arrow.
   d) She gets exactly 4 bull’s-eyes.
   e) She gets at least 4 bull’s-eyes.
   f) She gets at most 4 bull’s-eyes.

19. **Lefties redux.** Consider our group of 5 people from Exercise 17.

   a) How many lefties do you expect? b) With what standard deviation? c) If we keep picking people until we find a lefty, how long do you expect it will take?

20. **More arrows.** Consider our archer from Exercise 18.

   a) How many bull’s-eyes do you expect her to get? b) With what standard deviation? c) If she keeps shooting arrows until she hits the bull’s-eye, how long do you expect it will take?

21. **Still more lefties.** Suppose we choose 12 people instead of the 5 chosen in Exercise 17.

   a) Find the mean and standard deviation of the number of right-handers in the group.
   b) What’s the probability that i) they’re not all right-handed? ii) there are no more than 10 righties? iii) there are exactly 6 of each? iv) the majority is right-handed?

22. **Still more arrows.** Suppose our archer from Exercise 18 shoots 10 arrows.

   a) Find the mean and standard deviation of the number of bull’s-eyes she may get.
   b) What’s the probability that i) she never misses? ii) there are no more than 8 bull’s-eyes? iii) there are exactly 8 bull’s-eyes? iv) she hits the bull’s-eye more often than she misses?

23. **Vision.** It is generally believed that nearsightedness affects about 12% of all children. A school district tests the vision of 169 incoming kindergarten children. How many would you expect to be nearsighted? With what standard deviation?
24. **International students.** At a certain college, 6% of all students come from outside the United States. Incoming students are assigned at random to freshman dorms, where students live in residential clusters of 40 freshmen sharing a common lounge area. How many international students would you expect to find in a typical cluster? With what standard deviation?

25. **Tennis, anyone?** A certain tennis player makes a successful first serve 70% of the time. Assume that each serve is independent of the others. If she serves 6 times, what’s the probability she gets

   a) all 6 serves in?
   b) exactly 4 serves in?
   c) at least 4 serves in?
   d) no more than 4 serves in?

26. **Frogs.** A wildlife biologist examines frogs for a genetic trait he suspects may be linked to sensitivity to industrial toxins in the environment. Previous research had established that this trait is usually found in 1 of every 8 frogs. He collects and examines a dozen frogs. If the frequency of the trait has not changed, what’s the probability he finds the trait in

   a) none of the 12 frogs?
   b) at least 2 frogs?
   c) 3 or 4 frogs?
   d) no more than 4 frogs?

27. **And more tennis.** Suppose the tennis player in Exercise 25 serves 80 times in a match.

   a) What are the mean and standard deviation of the number of good first serves expected?
   b) Verify that you can use a Normal model to approximate the distribution of the number of good first serves.
   c) Use the 68–95–99.7 Rule to describe this distribution.
   d) What’s the probability she makes at least 65 first serves?

28. **More arrows.** The archer in Exercise 18 will be shooting 200 arrows in a large competition.

   a) What are the mean and standard deviation of the number of bull’s-eyes she might get?
   b) Is a Normal model appropriate here? Explain.
   c) Use the 68–95–99.7 Rule to describe the distribution of the number of bull’s-eyes she may get.
   d) Would you be surprised if she made only 140 bull’s-eyes? Explain.

29. **Apples.** An orchard owner knows that he’ll have to use about 6% of the apples he harvests for cider because they will have bruises or blemishes. He expects a tree to produce about 300 apples.

   a) Describe an appropriate model for the number of cider apples that may come from that tree. Justify your model.
   b) Find the probability there will be no more than a dozen cider apples.
   c) Is it likely there will be more than 50 cider apples? Explain.

30. **Frogs, part II.** Based on concerns raised by his preliminary research, the biologist in Exercise 26 decides to collect and examine 150 frogs.

   a) Assuming the frequency of the trait is still 1 in 8, determine the mean and standard deviation of the number of frogs with the trait he should expect to find in his sample.
   b) Verify that he can use a Normal model to approximate the distribution of the number of frogs with the trait.
   c) He found the trait in 22 of his frogs. Do you think this proves that the trait has become more common? Explain.

31. **Lefties again.** A lecture hall has 200 seats with folding arm tablets, 30 of which are designed for left-handers. The typical size of classes that meet there is 188, and we can assume that about 13% of students are left-handed. What’s the probability that a right-handed student in one of these classes is forced to use a lefty arm tablet?

32. **No-shows.** An airline, believing that 5% of passengers fail to show up for flights, overbooks (sells more tickets than there are seats). Suppose a plane will hold 265 passengers, and the airline sells 275 tickets. What’s the probability the airline will not have enough seats, so someone gets bumped?

33. **Annoying phone calls.** A newly hired telemarketer is told he will probably make a sale on about 12% of his phone calls. The first week he called 200 people, but only made 10 sales. Should he suspect he was misled about the true success rate? Explain.

34. **The euro.** Shortly after the introduction of the euro coin in Belgium, newspapers around the world published articles claiming the coin is biased. The stories were based on reports that someone had spun the coin 250 times and gotten 140 heads—that’s 56% heads. Do you think this is evidence that spinning a euro is unfair? Explain.

35. **Seatbelts II.** Police estimate that 80% of drivers now wear their seatbelts. They set up a safety roadblock, stopping cars to check for seatbelt use.

   a) How many cars do they expect to stop before finding a driver whose seatbelt is not buckled?
   b) What’s the probability that the first unbelted driver is in the 6th car stopped?
   c) What’s the probability that the first 10 drivers are all wearing their seatbelts?
   d) If they stop 30 cars during the first hour, find the mean and standard deviation of the number of drivers expected to be wearing seatbelts.
   e) If they stop 120 cars during this safety check, what’s the probability they find at least 20 drivers not wearing their seatbelts?

36. **Rickets.** Vitamin D is essential for strong, healthy bones. Our bodies produce vitamin D naturally when sunlight falls upon the skin, or it can be taken as a dietary supplement. Although the bone disease rickets was largely eliminated in England during the 1950s, some people there are concerned that this generation of children is at increased risk because they are more likely to watch TV or play computer games than spend time outdoors. Recent research indicated that about 20% of British children are deficient in vitamin D. Suppose doctors test a group of elementary school children.
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a) What’s the probability that the first vitamin D–deficient child is the 8th one tested?
b) What’s the probability that the first 10 children tested are all okay?
c) How many kids do they expect to test before finding one who has this vitamin deficiency?
d) They will test 50 students at the third-grade level. Find the mean and standard deviation of the number who may be deficient in vitamin D.
e) If they test 320 children at this school, what’s the probability that no more than 50 of them have the vitamin deficiency?

37. ESP. Scientists wish to test the mind-reading ability of a person who claims to “have ESP.” They use five cards with different and distinctive symbols (square, circle, triangle, line, squiggle). Someone picks a card at random and thinks about the symbol. The “mind reader” must correctly identify which symbol was on the card. If the test consists of 100 trials, how many would this person need to get right in order to convince you that ESP may actually exist? Explain.

38. True-False. A true-false test consists of 50 questions. How many does a student have to get right to convince you that he is not merely guessing? Explain.

39. Hot hand. A basketball player who ordinarily makes about 55% of his free throw shots has made 4 in a row. Is this evidence that he has a “hot hand” tonight? That is, is this streak so unusual that it means the probability he makes a shot must have changed? Explain.

40. New bow. Our archer in Exercise 18 purchases a new bow, hoping that it will improve her success rate to more than 80% bull’s-eyes. She is delighted when she first tests her new bow and hits 6 consecutive bull’s-eyes. Do you think this is compelling evidence that the new bow is better? In other words, is a streak like this unusual for her? Explain.

41. Hotter hand. Our basketball player in Exercise 39 has new sneakers, which he thinks improve his game. Over his past 40 shots, he’s made 32—much better than the 55% he usually shoots. Do you think his chances of making a shot really increased? In other words, is making at least 32 of 40 shots really unusual for him? (Do you think it’s his sneakers?)

42. New bow, again. The archer in Exercise 40 continues shooting arrows, ending up with 45 bull’s-eyes in 50 shots. Now are you convinced that the new bow is better? Explain.

JUST CHECKING
Answers

1. There are two outcomes (contact, no contact), the probability of contact is 0.76, and random calls should be independent.

2. Binomial, with \( n = 1000 \) and \( p = 0.76 \). For actual calculations, we could approximate using a Normal model with \( \mu = np = 1000(0.76) = 760 \) and \( \sigma = \sqrt{npq} = \sqrt{1000(0.76)(0.24)} \approx 13.5 \).

3. Geometric, with \( p = 0.29 \).